No. of Printed Pages : 5

Sem-I-Math-CC-1
(Reg&Back)

2021

Time - 3 hours

Full Marks - 60

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
The symols used have their usual meaning.
Candidates are required to answer
in their own words as far as practicable.

GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 8

(a) Find $\frac{dy}{dx}$ when $y = \sin h(4x - 8)$.

(b) A function f is concave up on (a, b) if f' is ________ on (a, b).

(c) Express as Riemann integration of limit of sum

$$\frac{n^2}{(n^2+1)^{\frac{3}{2}}} + \frac{n^2}{(n^2+2^2)^{\frac{3}{2}}} + \frac{n^2}{(n^2+3^2)^{\frac{3}{2}}} + \dots\dots\dots + \frac{n^2}{\{n^2+(n-1)^2\}^{\frac{3}{2}}}$$

(d) What is the value of $\int_0^{\pi/2} \log \tan x \, dx$?

P.T.O.

(e) What is rectification ?

(f) If the eccentricity of a conic is less than one, what is the name of the conic ?

(g) If **u**, **v** and **w** lies in the same plane, then find the value of **u** . (**v** × **w**).

(h) Evaluate $\lim_{t \to 0^+} \left(2\sqrt{t}\mathbf{i} + \frac{\sin t}{t}\mathbf{j}\right)$.

GROUP – B

2. Answer any eight of the following questions. [1½ × 8

(a) What is the nth derivative of $\frac{d^n}{dx^n}(\sin(ax+b))$?

(b) Determine $\lim_{x \to \frac{\pi}{2}} \frac{\tan 3x}{\tan x}$.

(c) Evaluate $\int_0^{\pi/2} \cos^6 2t\, dt$.

(d) Evaluate $\int_0^a \frac{x^4}{\sqrt{a^2 - x^2}}\, dx$.

(e) Find the asymptote parallel to the coordinate axes of the curve

$$\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1.$$

(f) Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval [1, 4] is revolved about the x-axis.

(g) Rotate the coordinate axes to remove the xy term of $xy = -9$.

(h) Find equation for the ellipse with foci $(0, \pm 2)$ and major axis with endpoints $(0, \pm 4)$.

(i) Find the natural domain of

$$r(t) = \langle \ln(t-1), e^t, \sqrt{t} \rangle.$$

(j) Find a vector equation of the line tangent to the graph of

$$r(t) = (3t - 1)\mathbf{i} + \sqrt{3t + 4}\,\mathbf{j} \text{ at } P_0(-1, 2).$$

GROUP – C

3. Answer any eight questions. [2 × 8

(a) If $y = \sin(\sin x)$, prove that

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0.$$

(b) Find the asymptote of the hyperbolic spiral $r\theta = a$.

(c) Find the points of inflexion on the curve $y = (\log x)^3$.

(d) Find the value of $\int_0^{\pi/6} \cos^6 3\theta \sin^2 6\theta \, d\theta$.

(e) Evaluate $\int_0^{\pi/2} \frac{dx}{4 + 5 \sin x}$.

(f) The loop of the curve $2ay^2 = x(x - a)^2$ revolves about x-axis. Find the volume of the solid so generated.

(g) Find the whole length of the astroid

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}.$$

(h) Find the equation of the hyperbola with vertices $(0, \pm 8)$ and asymptotes $y = \pm \frac{4}{3} x$.

(i) A particle moves along a circular path in such a way that its x and y-coordinates at time t are $x = 2 \cos t$, $y = 2 \sin t$. Find the instantaneous velocity of the particle at time t.

(j) Suppose that a particle moves through 3-spaces so that its position vector at time t is

$$r(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}.$$

Find the scalar tangential components of acceleration at time t.

GROUP – D

*Answer **any four** questions.*

4. If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that [6

$$x^2 y_{n+2} + (2n + 1)xy_{n+1} + 2n^2 y_n = 0.$$

5. Trace the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$. [6

6. Find the asymptotes of the cubic curve [6

$$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0.$$

7. Evaluate : $\int \frac{d\theta}{(a \sin^2 \theta + b \cos^2 \theta)^2}$. [6

8. Rectify the curve $x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$. [6

9. The loop of the curve $2ay^2 = x(x - a)^2$ revolves about x-axis. Find the volume of the solid so generated. [6

10. Show that the graph of the given equation is an ellipse. Find its foci, vertices and the ends of its minor axis. [6

2021

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GROUP – A

1. Answer <u>all</u> questions as directed. [1 × 12

(a) How many subsets of {a, b, c, d} atr there altogether ?

(b) If a relation R is not symmetric, then it is antisymmetric. (Write true or false.)

(c) "⇔" defines an equivalence relation. (Write true or false.)

(d) The relation "<" on the set of real numbers is a partial order. (Write true or false.)

(e) The first step in the principle of mathematical induction is to check the n = 1 case. (Write true or false.)

(f) The sequence 1, 1, 1, ... is both an arithmetic and geometric sequence. (Write true or false.)

P.T.O.

(g) If $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$, then $(A^2)^T =$ ________.

(Fill in the blank.)

(h) The adjoint matrix of $A=\begin{pmatrix}2 & 1\\ 3 & 6\end{pmatrix}$ is ________.

(Fill in the blank.)

(i) Thed order of highest non-zero minor of a matrix is called ________. (Fill in the blank.)

(j) If graphs G_1 and G_2 have the same number of vertices, then G_1 and G_2 are isomorphic. (Write true or false.)

(k) A Hamiltonian path in graph is a path that passes through every ________ (exactly one). (Fill in the blank.)

(l) For which n is K_n planar ?

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) Write the contrapositive of the implication "If x is an even number, then $x^2 + 3x$ is an even number".

(b) Write the negation of $p \Rightarrow q$.

(c) Write Fermat's little theorem (statement only).

(d) If $A \subset B$, then prove $B' \subset A'$.

(e) If ${}^{10}C_r = {}^{10}C_7$, find possible values of r.

(f) A man, woman, boy, girl, dog and cat are walking down a long and winding road one after other. In how many ways can this happen if the dog immediately follows the boy ?

(g) If an inverse of a matrix A exists, then prove that it is unique.

(h) Evaluate

$$\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$$

(i) Define bipartite graph.

(j) Define a subgraph.

GROUP – C

3. Answer <u>any eight</u> questions [3 × 8

(a) For sets A and B, prove that $A \cap B = A$ if and only if $A \subseteq B$.

(b) Let A = {1, 2, 3, 4, 5, 6}, let S = P(A), for a, b $\in$ S defined a ~ b if a and b have the same number of elements. Prove ~ defines an equivalence relation.

(c) Suppose a, b, x are integers such that a | bx. If and b are relatively prime, then prove a | x.

(d) Prove that 3^{2n-1} is divisible by 8 for every $n \geq 1$.

(e) Given five points inside a square whose side have length 2. Prove that two are within $\sqrt{2}$ of each other.

(f) Using the Binomial theorem, expand $\left(x-\frac{4}{x}\right)^6$ and simplify.

(g) Determine A^{-1} for given $A=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$.

(h) Without expanding, prove that

$$\begin{vmatrix} x & x+3 & x+6 \\ x+1 & x+4 & x+7 \\ x+2 & x+5 & x+8 \end{vmatrix} = 0.$$

(i) Prove that a graph that contains a triangle cannot be bipartite.

(j) Prove that for any $n \geq 4$ two isomorphic graphs must contain the same number of n-cycles.

GROUP – D

*Answer **any four** questions.*

4. Prove that [7

$$[\sim(p \leftrightarrow q)] \Leftrightarrow [\sim(p \wedge (\sim q)) \vee (q \wedge (\sim p))].$$

5. State and prove division algorithm. [7

6. Solve $a_n = -2a_{n-1} + 3a_{n-2} + 6^n,\ n \geq 2$,
given $a_0 = -1,\ a_1 = 5$. [7

7. How many integers between 1 and 300 (inclusive) are divisible by at least one of 3, 5, 7 ? [7

8. Find the rank of the following matrix : [7

$$\begin{bmatrix} 1 & 3 & 2 \\ -1 & 7 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

9. Determine the eigen values and corresponding eigen vectors for the following matrix : [7

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

10. Let G be a graph with $n \geq 4$ vertices each of degree at least $\frac{n}{2}$. Show that G contains a 4-cycle. [7

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GROUP – A

1. Answer all questions and fill in the blanks as required. [1 × 8

(a) What is the relation between curvature and radius of curvature ?

(b) When two spheres are touched to each other ?

(c) What is Rolle's theorem ?

(d) Evaluate $\lim_{x \to 0} \frac{\tan 3x}{2x}$.

(e) Find the first order partial derivative if $f(x, y) = e^{an} \sin by$.

(f) $\lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$ = ________.

(g) Find the order of the differential equation when the general solution is $y = mx + c$.

(h) Find the integrating factor of the differential equation

$$x \frac{dy}{dx} + y = 2x^2.$$

GROUP – B

2. Answer any eight of the following questions. [1½ × 8

(a) Find the radius of curvature of $y = c \cos h \frac{x}{c}$.

(b) Find centre and radius of

$$2x^2 + 2y^2 + 2z^2 + x + y + z + 1 = 0.$$

(c) Show that $f(x) = | x |$ at $x = 0$ is continuous.

(d) What is the Maclaurin's series of $\cos x$?

(e) Verify Rolle's theorem $f(x) = x^2$ in $[-1, 1]$.

(f) If $u = \tan^{-1} \frac{x^3 + y^3}{x + y}$, then show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u.$$

(g) What is a homogeneous function and give an example.

(h) Solve : $\frac{dy}{dx} = e^y e^x$.

(i) Elimination of arbitrary function and form the partial differential equation of

$$z = x + y + f(x, y).$$

(j) Find the general solution of $y'' + 8y = 0$.

GROUP – C

3. Answer <u>any eight</u> questions [2 × 8

(a) Find the asymptotes of $x^3 + y^3 - 3axy = 0$.

(b) Trace the curve of $x^2y^2 = x^2 - a^2$.

(c) Find the equation of the sphere when it passes through (0, 0, 0), (0, 1, –1), (–1, 2, 0), (1, 2, 3).

(d) Test the differentiability of $f(x) = [x]$ at $x = 2$.

(e) Evaluate $\lim_{x \to 0} (1 + x)^{\frac{1}{x}}$.

(f) Verify that if $z = \tan^{-1}\left(\frac{y}{x}\right)$, then

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

(g) Prove that $f_{xy} \neq f_{yx}$ at origin when

$$f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0). \end{cases}$$

(h) Find the extremum of $f(x) = (x^2 - 3x + 1)e^{-x}$.

(i) Solve : $(x^2 + y^2 + 2x)dx + 2y\,dy = 0$.

(j) Solve : $(x^2D^2 - xD + 2)y = x \log x$ where $D = \frac{d}{dx}$.

GROUP – D

Answer **any four** *questions.*

4. Prove that $\rho = \frac{a^2b^2}{p^3}$ when $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

'p' is the perpendicular from the centre on the tangent at the point (x, y). [6

5. Find the equation of sphere through (1, 0, 0), (0, 1, 0), (0, 0, 1) with its centre on the plane $3x - y + z = 2$. [6

6. Write the ststement of Taylor's series and prove it. [6

7. Discuss the minimum and maximum of [6

$$f(x, y) = \sin x + \sin y + \sin(x + y).$$

8. Solve : $(x^2D^2 + xD - 4)y = x^2$. [6

9. Solve : $(D^2 + 1)y = \operatorname{cosec} x$. [6

10. Solve : [6

$$x^2y'' - (x^2 + 2x)y' + (x + 2)y = x^3e^x.$$